Explicitly Valuing Mathematical Creativity in Proof-Based Courses

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MATHEMATICS

The Creativity Research Group

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The Scene in 2016

- Wolfram Alpha
- Web sites dedicated to textbook solutions
- Automatic theorem-provers
- Workforce change in the United States
  - More than 1,500 Chief Executive Officers from 60 countries and 33 industries worldwide, believe that -- more than rigor, management discipline, integrity or even vision -- successfully navigating an increasing complex world will require **creativity**. (IBM 2010 Global CEO Study)

Creativity in Guidelines & Standards

MAA CUPM 2015 Guidelines

- A successful major offers a program of courses to gradually and intentionally leads students from basic to advanced levels of critical and analytical thinking, ***while encouraging creativity*** and ***excitement about mathematics.***
- Major programs should include activities designed to promote students' progress in learning to approach mathematical problems with curiosity and ***creativity*** and ***persist*** in the face of difficulties.

Valuing Creativity

"It is in the best interest of the field of mathematics education that we identify and nurture creative talent in the mathematics classroom" (Sriraman, 2004, p. 32).

"For it is through mathematical creativity that we see the essence of what it means to 'do' and learn mathematics." (Liljedahl, 2009, p. 239).

Defining Creativity

What is mathematical creativity?

In particular, in proving?

Over 100 definitions of mathematical creativity!
(Mann, 2006)

Defining Creativity

- Psycho-Analytic: Many mathematicians describe an **enlightenment** that is somewhat **unexpected** (Hadamard, 1945; Poincare, 1958).
- Product: Some focus on emphasizing whether the end-product is **original** and **useful** (Runco & Jaeger, 2012), perhaps to the mathematics field (Csikszentmihalyi, 1988).
- Process: describe it as a process that involves **different modes of thinking**, some of **an unusual** nature (Balka, 1974).

Absolute vs. Relative Creativity

- Absolute creativity versus Relative creativity
  - Historical inventions or discoveries at a global level
  - The discoveries by a specific person within a specific reference group, to human imagination that creates something new (Vygotsky, 1982, 1984)

K-12 Math Creativity

- Torrance (1966, 1978) created testing for creativity and giftedness in K-12 education
- Silver (1997) expanded three aspects of K-12 mathematical creativity
  - Flexibility - An ability to look at a problem from new perspective
  - Originality - Using an unexpected or unusual approach
  - Fluency - Applying ideas, tools of one area in a different area
- Elaboration – Expanding on each approach (Torrance, 1978)
- Iconoclasm – Affect, Self-efficacy of each approach (Chamberlain & Mann, 2015)

Situating Our Work

- Creativity in K-12 classrooms is different than the kind employed by mathematicians (Sriraman, 2005)
- Proof-writing aligns more with mathematicians' creativity

Overarching Research Question

How can we **explicitly value** and **foster potential for** undergraduate students' creativity in mathematics?

Specific Research Questions

- What properties/actions can undergraduate students learn/enact to generate potential for being creative in proving?
- How can we implement said properties/actions in the classroom?

Creativity-in-Progress Rubric (CPR) on Proving

- Creativity rubric from AAC&U (Rhodes, 2010)
- Leikin's (2009) Problem-Solving Rubric
- Interview with mathematicians (Tang, et al., 2015)
- Constant alpha-testing on students' LiveScribe work
- Feedback from past presentations
- Coding student process with mathematicians

CPR on Proving (cont.)

- ***Categories***

  (1) Making Connections

  (2) Taking Risks

- ***Levels (Continuum)***
  - Beginning
  - Developing
  - Advancing

CPR on Proving

MAKING CONNECTIONS:	Beginning	Developing	Advancing
Between Definitions/Theorems	Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving	Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving	Implements definitions/theorems from the course and/or other resources outside the course in their proving
Between Representations	Provides a representation with no attempts to connect it to another representation	Recognizes connections between representations	Uses connections between different representations
Between Examples	Generates one or two specific examples with no attempt to connect them	Recognizes a connection between the generated examples	Uses the key idea synthesized from generating examples

CPR on Proving

TAKING RISKS:	**Beginning**	**Developing**	**Advancing**
Tools and Tricks[2]	Uses a tool or trick that is algorithmic or conventional for the course or the student	Uses a tool or trick that is model-based or partly unconventional[3] for the course or the student	Creates a tool or trick that is unconventional for the course or the student
Flexibility[4]	Begins a proof attempt (or more than one proof attempt), but uses only one approach	Acknowledges and/or uses more than one proving approach, but only draws on one proof technique	Uses more than one proof technique
Posing Questions	Recognizes there should be a question asked, but does not pose a question[5]	Poses questions clarifying a statement of a definition or theorem	Poses questions about reasoning within a proof
Evaluation of Proof Attempt	Examines surface-level[6] features of a proof attempt	Examines an entire proof attempt for logical or structural flow	Examines and *revises* an entire proof attempt for logical or structural flow

What the CPR on Proving is and is not

- It is NOT assessing "correctness" or "validity" of the final proof.
- It is examining the full process of proof production.
- It is NOT a rubric to label student's creativity!
- It makes explicit some aspects that may promote mathematical creativity.

Fostering Creativity

- Non-judgmental environment
- Authority in creativity
- "To promote creativity and mathematical thinking, teachers should encourage good ideas even (and, in fact, especially) when a student suggests an unexpected answer or when the answers are inaccurate." (Hershkovitz, Peled, & Littler, 2009, p. 265)

Fostering Creativity (cont.)

- Problem Posing/Conjecturing
- Solutions as problems: "Students are rarely asked to view a solution to a problem as a starting point in problem solving" (Knuth, 2002, p. 129)

Creativity in Business

- The organizational climates that stimulate creativity (Amabile, 1988; Isaksen, 1995):
  - feel challenged by their goals, operations and tasks
  - feel able to take initiatives and to find relevant information
  - feel able to interact with others
  - feel that new ideas are met with support and encouragement
  - feel able to put forward new ideas and views
  - experience much debate within a prestige-free and open environment
  - feel uncertainty is tolerated and thus risk-taking is encouraged.

Recap of the Research

- Two major actions that create potential for mathematical creativity:
  - Making Connections
  - Taking Risks
- Our conjecture is that, if both are fostered in any environment, creative (and eventually valid) products will be produced.

Future Research

- Implementation of CPR on Proving in the classroom
- Creativity x Neuroscience
  - EEG "reliving" with the proving process
- CPR on Problem Solving
- Creativity in a social setting
- CPR on Linear Algebra, Calculus, and Pre-Calc?!?

It must not be forgotten that the basic law of children's creativity is that its value lies not in its results, not in the product of creation, but in the process itself. It is not important what children create, but that they do create, that they exercise and implement their creative imagination.
(Vygotsky, 2004, p. 72)

Thank you!

Questions?

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