CPR on Proving: Explicitly Valuing Mathematical Creativity

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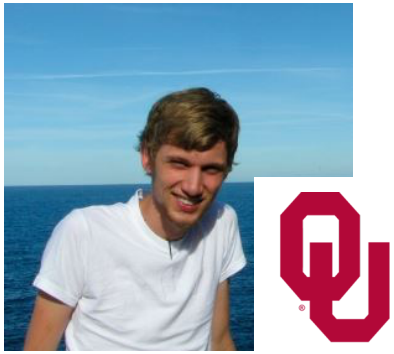
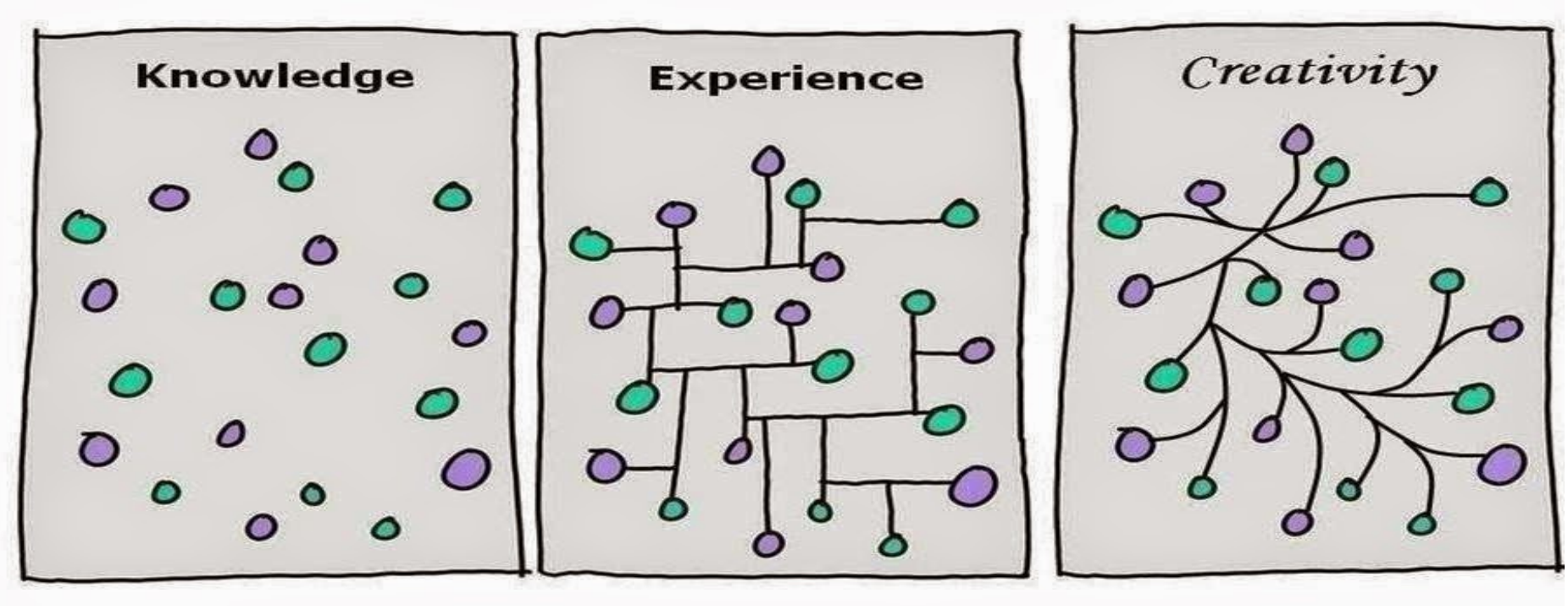
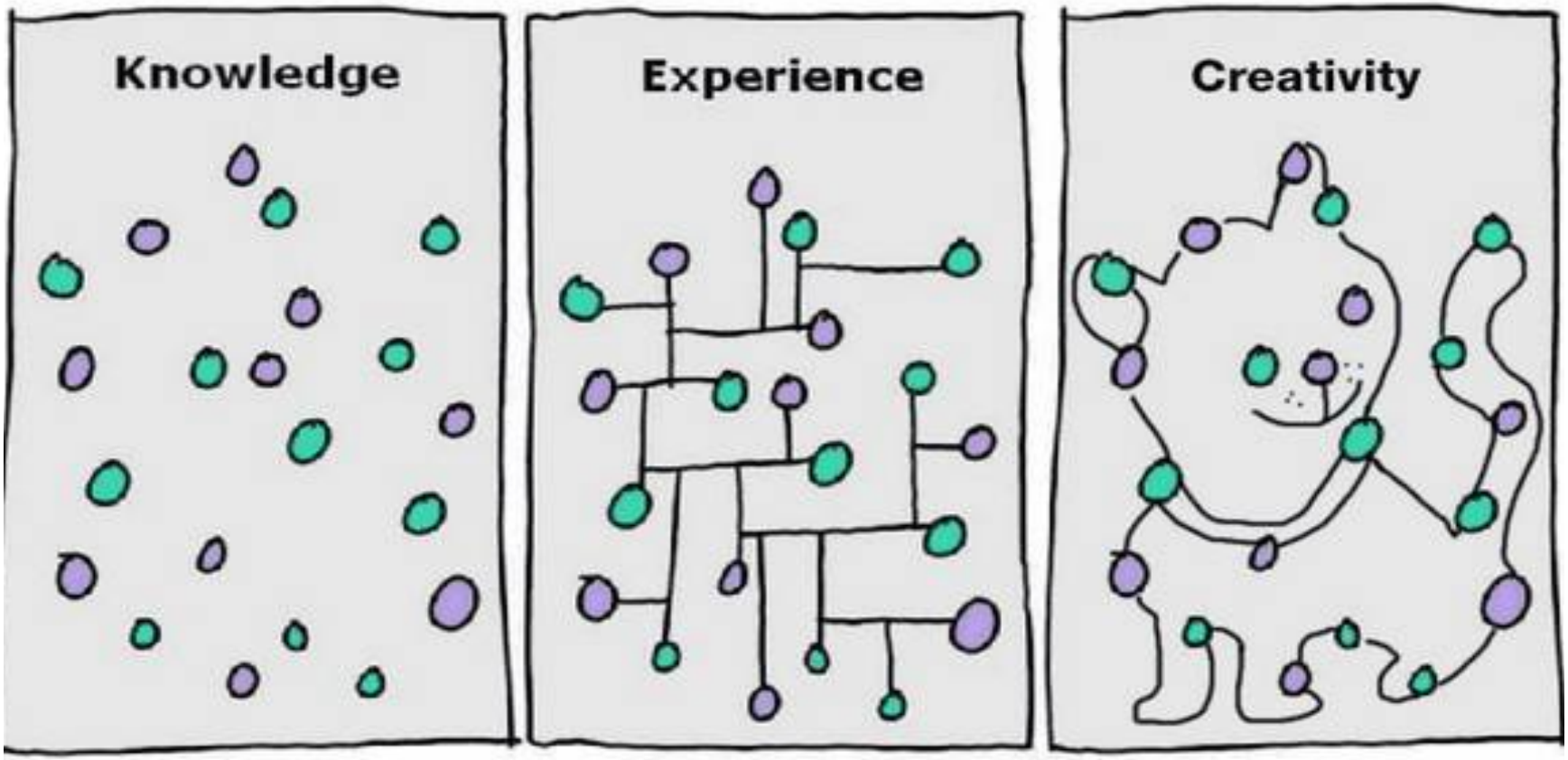


What is Creativity?





What is *Mathematical* creativity?

- ***Statement:*** If n is an integer such that $n \geq 3$, then $n^3 \geq (n+1)^2$.

Three proofs - for each proof decide (using your definition) if it is creative or not…

Definitions and Perspectives

- Psycho-Analytic: Many mathematicians describe an **enlightenment** that is somewhat **unexpected** (Hadamard, 1945; Poincare, 1958).
- Product: Some focus on emphasizing whether the end-product is **original** and **useful** (Runco & Jaeger, 2012), perhaps to the mathematics field (Csikszentmihalyi, 1988).
- Process: Describe it as a process that involves **different modes of thinking**, some of **an unusual** nature (Balka, 1974).

Definitions and Perspectives

- Creativity in K-12 classrooms is **different** than the kind employed by **[experts]** and that students' creativity needs to be evaluated according to **their prior experiences.** (Sriraman, 2005)

Definitions and Perspectives

- Absolute creativity versus Relative creativity
  - Historical inventions or discoveries at a global level
  - The discoveries by a specific person within a specific reference group, to human imagination that creates something new (Vygotsky, 1982, 1984)

Definitions and Perspectives

- Our “working” definition by Sriraman (2015):

Creativity refers to anything someone ***does*** in a way that is ***original to the creator*** and that is appropriate to the purpose or goal of the creator [in a given task].

Why Mathematical Creativity?

The Scene in 2017

- Wolfram Alpha
- Web sites dedicated to textbook solutions
- Automatic theorem-provers

Valuing Creativity

More than 1,500 Chief Executive Officers from 60 countries and 33 industries worldwide, believe that -- more than rigor, management discipline, integrity or even vision -- successfully navigating an increasing complex world will require **creativity**.
(IBM 2010 Global CEO Study.)

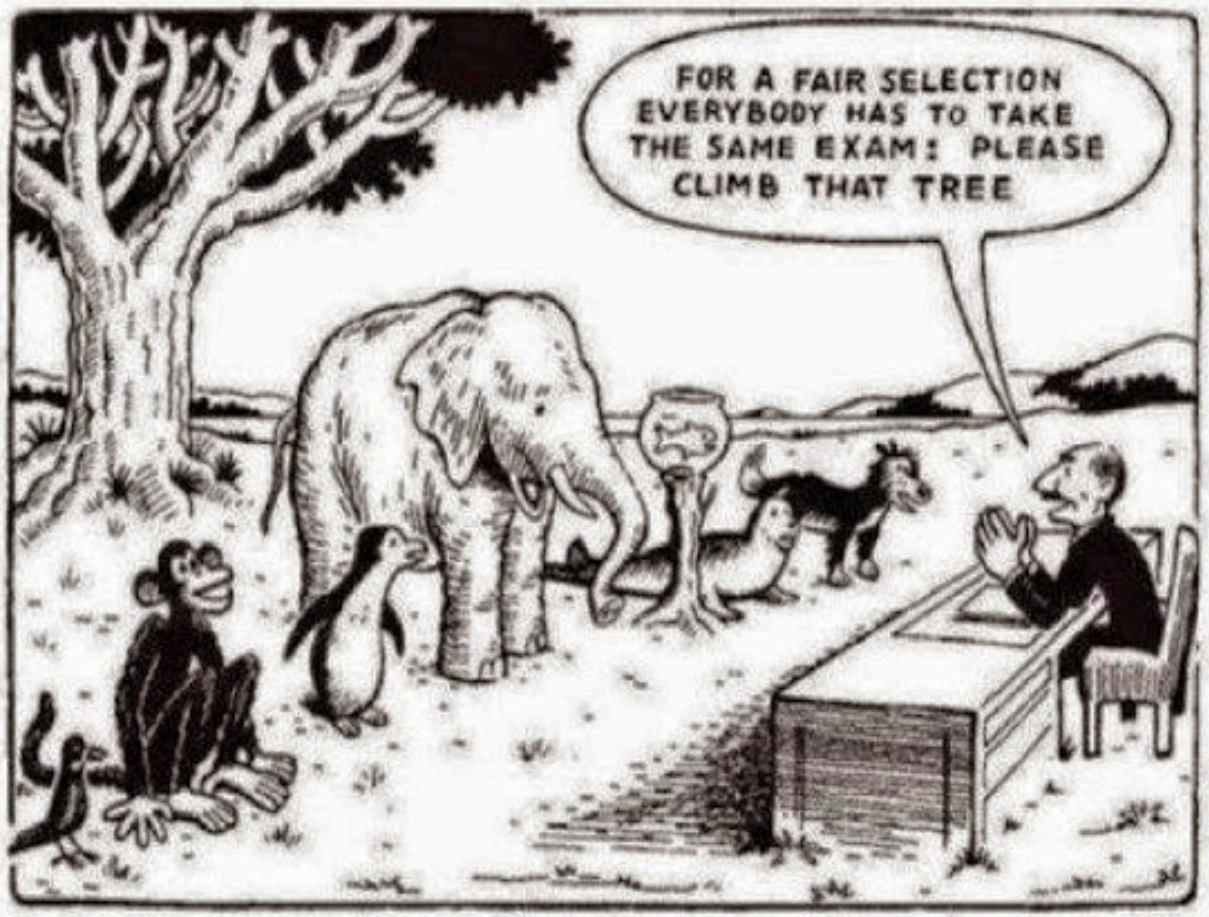
Valuing Creativity

[T]eaching engineers (and other STEM disciplines) to **think creatively** is absolutely essential to a society's ability to generate wealth, and as a result provide a stable, safe, healthy and productive environment for its citizens. (Cropley, 2015)

Valuing Creativity

MAA CUPM 2015 Guidelines

- A successful major offers a program of courses to gradually and intentionally leads students from basic to advanced levels of critical and analytical thinking, ***while encouraging creativity*** and ***excitement about mathematics.***
- Major programs should include activities designed to promote students' progress in learning to approach mathematical problems with ***curiosity*** and ***creativity*** and persist in the face of difficulties.



Our Education System

"Everybody is a genius. But if you judge a fish by its ability to climb a tree, it will live its whole life believing that it is stupid."

- Albert Einstein

Valuing Creativity

[I]n seeking to facilitate the development of talented young mathematicians, **neglecting** to recognize **creativity may drive** the creatively **talented underground or**, worse yet, cause them to **give up** the study of **mathematics** altogether. (Mann, 2005, p. 239)

How do we do it?

Creativity-in-Progress Rubric (CPR) on Proving

Development of CPR

- Interviews with research mathematicians
  - Are you creative?
- Literature guidance
    - Fluency
    - Flexibility
    - Originality
    - Elaboration
  - K-12 research studies
  - AAC&U Creativity Thinking Rubric

Creativity-in-Progress Rubric (CPR)

- ***Categories***

  (1) Making Connections (Fluency, Elaboration)

  (2) Taking Risks (Flexibility, Originality, Elaboration)

- ***Levels (Continuum)***
  - Beginning
  - Developing
  - Advancing

Making Connections

The ability to connect the proving task with definitions, theorems, multiple representations, and examples from the current course that a student is in, and possible prior experiences from previous courses.

Creativity-in-Progress Rubric (CPR)

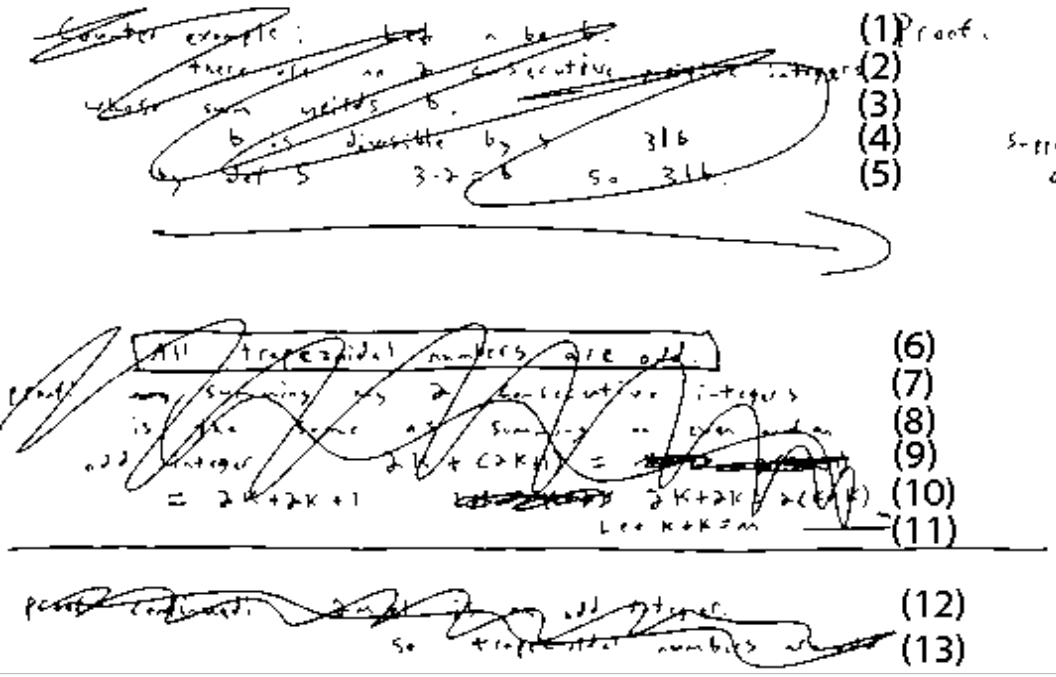
MAKING CONNECTIONS:	Beginning	Developing	Advancing
Between Definitions/Theorems	Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving	Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving	Implements definitions/theorems from the course and/or other resources outside the course in their proving
Between Representations	Provides a representation with no attempts to connect it to another representation	Recognizes connections between representations	Uses connections between different representations
Between Examples	Generates one or two specific examples with no attempt to connect them	Recognizes a connection between the generated examples	Uses the key idea synthesized from generating examples

Taking Risks

The ability to actively attempt a proof, demonstrate flexibility in using multiple approaches or techniques, posing questions about reasoning within the attempts, and evaluating those attempts.

Creativity-in-Progress Rubric (CPR)

TAKING RISKS:	Beginning	Developing	Advancing
Tools and Tricks[2]	Uses a tool or trick that is algorithmic or conventional for the course or the student	Uses a tool or trick that is model-based or partly unconventional[3] for the course or the student	Creates a tool or trick that is unconventional for the course or the student
Flexibility[4]	Begins a proof attempt (or more than one proof attempt), but uses only one approach	Acknowledges and/or uses more than one proving approach, but only draws on one proof technique	Uses more than one proof technique
Posing Questions	Recognizes there should be a question asked, but does not pose a question[5]	Poses questions clarifying a statement of a definition or theorem	Poses questions about reasoning within a proof
Evaluation of Proof Attempt	Examines surface-level[6] features of a proof attempt	Examines an entire proof attempt for logical or structural flow	Examines and *revises* an entire proof attempt for logical or structural flow



MAKING CONNECTIONS:	Beginning	Developing	Advancing
Between Definitions/Theorems			
Between Representations			
Between Examples			

TAKING RISKS:	Beginning	Developing	Advancing
Tools and Tricks			
Flexibility			
Perseverance			
Posing Questions			
Evaluation of the Proof Attempt			

Correctness

"I will risk it and say that **[a proof] doesn't have to be correct to be creative**. But at least it [the proof] should be fixable. It can happen that you have an original idea and you mess up details, which is not surprising **because if it is an original idea then it means that you haven't practiced that, [so] you would make mistakes**." (Dr. E- interview participant, 2013)

Definitions and Perspectives

- Our “working” definition by Sriraman (2015):

Creativity refers to anything someone ***does*** in a way that is ***original to the creator*** and that is appropriate to the purpose or goal of the creator [in a given task].

Creativity-in-Progress Rubric (CPR)

- It is NOT assessing “correctness” of the final product.
- It is for students to examine their own process of proof production.
- It is for teachers to create an environment for students to express their thinking.

Implementation of CPR

- Introductory proof course (whole semester)
- Elementary number theory course (in different parts)
- Combinatorics (elective) course (Homework assignments)

Students' Feedback

- [The rubric] lets me know that, you know, it's okay to go between examples, it's **ok to do this, it's ok to do that.**

- For example, if you have a proof, and you try a direct proof, well **try something different**! Do the contrapositive, or do the contradiction. You know, even if it may not work and in the end you spent an extra 20 or 30 minutes to do it, you know, it pays off in the end and it **builds your creativity**.

Students' Feedback

- [W]ell, I would kind of use [the CPR on Proving] as a **checklist** to go through it and when I'm **evaluating my proof**, I would use and say "could I **make any connection**?" …You know, but could I do more? Could I do it better? Could I **go from developing to advancing** in my proof?
- [W]hen **I got stuck** on the proof on a problem in the book, I would just look back to [the CPR on Proving], and 'oh let me try it this way, let me try it that way.

Students' Feedback

- I think it is sort of one of the bigger things that [CPR] has helped me do is, like, ***reflect on the sort of creative process*** that I have and ***it's kind of help me understand the ways that I can be mathematically creative,*** I guess.

Students' Feedback

- The **rubric itself helped kind of outline where you should go if you were lost**, um, in a very general sense. Um…and **the reflection process kind of, over time, helped to refine my problem solving strategies** so I kind of knew when I was spit balling, getting lost and kind of knew when, uh, I should probably keep attempting, uh, an idea because it was hopefully gonna pan out.

Students' Feedback

- What aspect of the course helped you to develop your creativity?

I think the biggest thing was ***being challenged in a way that I hadn't been before***. Essentially being challenged to, in these kinds of problems, not have a straight forward correct solution most of the time.

It must not be forgotten that the basic law of children's creativity is that its value lies not in its results, not in the product of creation, ***but in the process itself***. It is not important what children create, but that they do create, that they exercise and implement their creative imagination. (Vygotsky, in 2004 translation, p. 72)

THANK YOU!

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