Miloš Savić
University of Oklahoma
savic@ou.edu

Gülden Karakök
University of Northern Colorado
gulden.karakok@unco.edu

Gail Tang
University of La Verne
gtang@laverne.edu

Houssein El Turkey
University of New Haven
helturkey@newhaven.edu

Emilie Naccarato
University of Northern Colorado
emilie.naccarato@unco.edu

David Plaxco
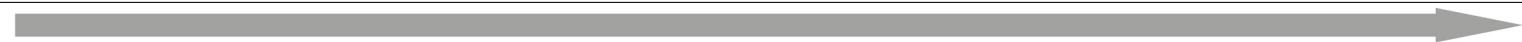University of Oklahoma
dplaxco@math.ou.edu

MAKING CONNECTIONS:

	Beginning	Developing	Advancing
Between Definitions/Theorems	Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving	Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving	Implements relevant definitions/theorems from the course and/or other resources outside the course in their proving
NA ☐			
Between Representations[1]	Provides a representation with no attempts to connect it to another representation	Provides multiple representations and recognizes connections between representations	Provides multiple representations and uses connections between different representations
NA ☐			
Between Examples	Generates one or two specific examples with no attempt to connect them	Generates one or two specific examples and recognizes a connection between them	Generates several specific examples and uses the key idea synthesized from their generation
NA ☐			

[1] We define a *mathematical representation* similar to NCTM's (2000) definition. It includes written work in the form of diagrams, graphical displays, and symbolic expressions. We also include linguistic expressions as a form of lexical or oral representation. For example, a student can use the lexical or oral representation, "the intersection of sets A and B"; a Venn Diagram to depict his/her mathematical thinking; a symbolic representation $A \cap B$; or set notation $\{x | x \in A \text{ and } x \in B\}$ (which is also a symbolic representation). Note the last two representations are in the same category, e.g. symbolic, but they are still considered two different representations.

TAKING RISKS:

	Beginning	**Developing**	**Advancing**
Tools and Tricks[2]	Uses a tool or trick that is algorithmic or conventional for the course or the student	Uses a tool or trick that is model-based or partly unconventional[3] for the course or the student	Creates a tool or trick that is unconventional for the course or the student
NA ☐	→		
Flexibility[4]	Begins a proof attempt (or more than one proof attempt), but uses only one approach	Acknowledges and/or uses more than one proving approach, but only draws on one proof technique	Uses more than one proof technique
NA ☐	→		
Posing Questions	Recognizes there should be a question asked, but does not pose a question[5]	Poses questions clarifying a statement of a definition or theorem	Poses questions about reasoning within a proof
NA ☐	→		
Evaluation of Proof Attempt	Examines surface-level[6] features of a proof attempt	Examines an entire proof attempt for logical or structural flow	Examines and *revises* an entire proof attempt for logical or structural flow
NA ☐	→		

[2] Based on the Originality category from Leikin (2009).

[3] Learned in a different context.

[4] A proof attempt is a continuous, sustained line of reasoning focused on a single theorem or conjecture. A proof approach is a proof attempt in which a new or different (to the prover) idea is introduced. Finally, a proof technique is a proof approach that addresses the overall logical structure of the proof. Common proof techniques include induction, proof by cases, direct proof, contradiction, and contrapositive.

[5] For example, a student writes a "?" next to something.

[6] Surface-level features include technical, computational, and line-to-line logical details.