What is Mathematical Creativity in Proving and How Can it be Fostered?

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April 1st, 2017

OU MATHEMATICS

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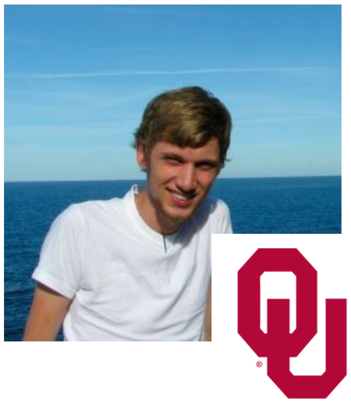
MATHEMATICS

The Creativity Research Group



Goal of "Talk"

Have everyone in the room **consider** mathematical creativity in their class

Consider how to implement one task/teaching idea from this talk about creativity

Why creativity?

- Mark Cuban, Feb. 17, 2017
  - "Cuban believes employers will soon be on the hunt for candidates who excel at creative and critical thinking." (http://www.inc.com/betsy-mikel/mark-cuban-says-this-will-soon-be-the-most-sought-after-job-skill.html)
- Wolfram Alpha, Slater/Chegg, Other Tech Innovations
- Creativity ⇒ Gain of content knowledge (Leikin, 2014)
- Creativity ⇒ Self-efficacy (Paul Regier, OU Grad Student)

Why creativity?

MAA CUPM 2015 Guidelines

- A successful major offers a program of courses to gradually and intentionally leads students from basic to advanced levels of critical and analytical thinking, ***while encouraging creativity*** and ***excitement about mathematics.***
- Major programs should include activities designed to promote students' progress in learning to approach mathematical problems with curiosity and ***creativity*** and ***persist*** in the face of difficulties.

But but but…

WHAT IS MATHEMATICAL CREATIVITY?

Over 100 definitions (Mann, 2006)

A process of offering **new solutions** or insights that are **unexpected** for the student, **with respect to his/her mathematics background** or the problems s/he has seen before (Sriraman & Liljedahl, 2006)

Unpacking that statement

- **A process…**
  - Not necessarily a/the end product
- **…of offering new solutions or insights that are unexpected…**
  - Originality and surprise
- **…for the student, with respect to his/her mathematics background or the problems s/he has seen before.**
  - Relative to the student instead of to his/her peers or mathematics in general

Example

- Here are four numbers:

$$15, 20, 23, 25$$

- Which number is different from the other three? Why?
- How many answers can you have?

Setting information

- 14 students (7 male, 7 female)
- Introduction-to-proof course
- Small university (~5,000 undergrad and grad students) in southwest US
- Instructor is a researcher in the group

How do we foster creativity?

- Tasks that give opportunities
- Instructors' practices and considerations
- Students' practices

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Example task

- Usual task on Calculus exam:
  Calculate the following integral $\int_1^2 x^2\, dx$
- Task that may promote creativity:
  What non-linear function satisfies the following:
  $\int_1^2 f(x)\, dx = 17$

Why does it "work?"

- Allows student to be mathematically flexible
  - Many correct answers
- Still tests the required skill
  - Student must know how to calculate an integral to verify that her/his solution is correct
- Can parlay this into discussion of other components of the definite integral OR the fundamental theorem of calculus

Example task 2

(a) (5 pts) Create a theorem where the result would contribute to the mathematical knowledge of our classroom community. OR create a theorem about something that you wondered about while working with sets. **You must use at least two sets.** Other symbols that you might use are "$=$", "$\subseteq$", "$\cap$", "$\cup$", and "$\emptyset$". Feel free to use power sets as well (+2 extra credit points if you do!).

(b) (5 pts) Prove your theorem.

Example task 2 cont.

- Power and authority of mathematics
- Students' freedom and (perhaps) identity
- Low entrance, high capacity

How to make such tasks?

- Problem-posing (Silver, 1997; Kwon, Park & Park, 2006)
  - Giving students the answer and letting students pose problems
- The "LEGO" approach
  - Giving students items and allowing students to construct their own mathematics
- Levenson (2013); Moore-Russo and Demler (under review); Zazkis and Holton (2009)

How do we foster creativity?

- Tasks that give opportunities
- **Instructors' practices and considerations**
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Question

What do you think instructors could do to foster mathematical creativity in the classroom?

Fostering creativity

- Mathematical creativity in undergraduate teaching
- From Zazkis and Holton (2009):
  - Learner-generated examples (Watson & Mason, 2005)
  - Counter-examples (Koichu, 2008)
  - Multiple solutions (Leikin 2007, 2009)
  - Changing parameters (Brown & Walter, 1983)
- "At the collegiate level, however, very little empirical research has yet described and analyzed the practices of teachers of mathematics" (Speer, Smith & Horvath, 2010, p. 99)

Five principles

- Sriraman (2005) conjectured five principles for fostering mathematical creativity in K-12 classrooms:
  - Gestalt
  - Aesthetic
  - Free Market
  - Scholarly
  - Uncertainty
- Savic et al. (accepted) – Examples in proof-based courses

Example task 2, cont.

Quiz 5: Conjectures created by class

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Are the following conjectures true or false? If true, prove. If false, show a counterexample **or discuss why it's false. If truth value cannot be determined because there is missing information, then you may add that missing information. For example, you may additional assumptions. You may also change symbols. For example if it's not true for the two sets to be equal, but it is if one is a subset of each other, then you can change "$=$" to "$\subseteq$".**

1. **Conjecture 1.** Let S and T be sets. Then $P(S) \setminus P(T) \subseteq P(S \setminus T)$.
2. **Conjecture 2.** Let A and B be sets. The $P((A \cap B)^c) = P(A^c \cup B^c)$.
3. **Conjecture 3.** Let A and B be sets. If $A \subseteq B$ and $B \subseteq C$ then $P(C^c) \subseteq P(A^c)$.
4. **Conjecture 4.** Let A and B be sets. If $A \subseteq B$, then $A \subseteq P(B)$.
5. **Conjecture 5.** Let S and T be sets. If $S \subseteq T$, then $T^c \subseteq S^c$.

Environmental Aspects

- Psychological safety (Rogers, 1962; Starko, 2013)
  - "Acceptance of the individual as having unconditional worth is at the core of psychological safety. This type of acceptance means that whatever his or her current condition, the person is seen as having value and potential." (Starko, 2013, p. 272)
- Creativity, Inquiry, and Equity (Tang et al., accepted)

Example of psych safety

- Instructor passed out copies of solutions to all students, and stated this:

  "...That's the exam 2 'solutions' and I say solutions in quotes because they're not all 100% correct, okay, but it doesn't matter. You know there are still really good ideas in there and that's what I want you to see."

How do we foster creativity?

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Question

- What are the roles that students have in the fostering of creativity in the classroom (save for the fact that they generate answers to problems)?

Students' Perspectives

- Metacognition (Schoenfeld, 1992)
  - Definition of creativity as a process, and students realizing that process
  - "Creative actions might benefit from meta-cognitive skills and vice versa, regarding the knowledge of one's own cognition and the regulation of the creative process" (Katz and Stupel, 2015, p. 69)

CPR on Proving

- Creativity-in-Progress Rubric (CPR) on Proving (Savic et al., 2016; Karakok et al., 2016)
- Focused on fostering creativity in the proving process
- Two major categories:
  - Making Connections
  - Taking Risks
- Created with the intention as a research tool, then teaching tool, and now a student/teaching tool
- El Turkey et al., (under review); Omar et al. (submitting in a week)

CPR on Proving

MAKING CONNECTIONS:

	Beginning	**Developing**	**Advancing**
Between Definitions/Theorems	Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving	Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving	Implements relevant definitions/theorems from the course and/or other resources outside the course in their proving
Between Representations[1]	Provides a representation with no attempts to connect it to another representation	Provides multiple representations and recognizes connections between representations	Provides multiple representations and uses connections between different representations
Between Examples	Generates one or two specific examples with no attempt to connect them	Generates one or two specific examples and recognizes a connection between them	Generates several specific examples and uses the key idea synthesized from their generation

CPR on Proving

TAKING RISKS:

	Beginning	**Developing**	**Advancing**
Tools and Tricks[2]	Uses a tool or trick that is algorithmic or conventional for the course or the student	Uses a tool or trick that is model-based or partly unconventional[3] for the course or the student	Creates a tool or trick that is unconventional for the course or the student
Flexibility[4]	Begins a proof attempt (or more than one proof attempt), but uses only one approach	Acknowledges and/or uses more than one proving approach, but only draws on one proof technique	Uses more than one proof technique
Posing Questions	Recognizes there should be a question asked, but does not pose a question[5]	Poses questions clarifying a statement of a definition or theorem	Poses questions about reasoning within a proof
Evaluation of Proof Attempt	Examines surface-level[6] features of a proof attempt	Examines an entire proof attempt for logical or structural flow	Examines and *revises* an entire proof attempt for logical or structural flow

Example of metacogntion

- And so I felt like, I felt like once the rubric was given to us, um, I was able to get a better idea – **not so much where I felt the instructor wanted me to be. Where I knew I wanted to be.** ..we had to like apply the rubric to one of our work. And before the assignment was even given out to us, like I already just started doing it and thinking 'I'm more at the beginning level. Like I get stuck I'm not trying a different [proving] method. I'm not, I just get stuck'. And so I felt like the rubric was really helpful in that it showed me where did I want to be. Like how do I get to the next level or the next phase? **And how do I get to advancing? And, like it gave me more guidance.**

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Conclusion

What will you consider about mathematical creativity that you did not know previously?

What did you take out of this talk that you will implement in your classrooms next week?

"So I think that like everyone's capable of mathematical creativity. I think that mathematical creativity is not really kind of taught or not made accessible to people, so I think people a lot of times don't realize that they're capable of being creative."

Thank you!

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