Research In The Classroom: Leveling the Playing Field

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Breaking Boundaries Conference

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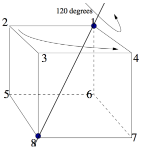
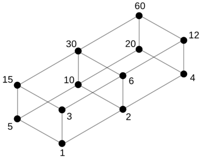
Teaching Challenges

- Providing a learning experience that offers the same level and depth of challenge to every student regardless of background or experience.
- Using a course I teach as a vehicle for developing skills in technical writing and prose.

Combinatorics - Math 106





- Terminal course: mostly juniors
- Has proof-based course as a prerequisite
- Math and computer science majors

The Beginning

Gail Tang, University of La Verne

CCMS Colloquium: “The Creativity Rubric”

Rubric

Creativity in Progress Rubric (CPR) on Proving

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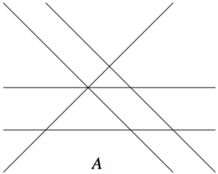
MAKING CONNECTIONS:	**Beginning**	**Developing**	**Advancing**
Between Definitions/Theorems	Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving	Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving	Implements relevant definitions/theorems from the course and/or other resources outside the course in their proving
Between Representations[1]	Provides a representation with no attempts to connect it to another representation	Provides multiple representations and recognizes connections between representations	Provides multiple representations and uses connections between different representations
Between Examples	Generates one or two specific examples with no attempt to connect them	Generates one or two specific examples and recognizes a connection between them	Generates several specific examples and uses the key idea synthesized from their generation

Rubric

Making Connections:
Between Definitions/Theorems

Beginning	Developing	Advancing
Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving	Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving	Implements relevant definitions/theorems from the course and/or other resources outside the course in their proving

Portfolio Problem



Determine the number of bounded and unbounded regions of a A-arrangement in terms of its intersection poset in as many of the following situations as you like:

1. $\mathcal{A}$ consists of V's in the plane; 3-space.
2. $\mathcal{A}$ consists of circles, no pair of which are tangent.
3. other geometric objects (example, varieties)

ENCOURAGING EXPERIMENTATION

- In proof-based courses, students often seek 'aha' moment
- In practice of math research, often conjectures are made based on evidence after extensive experimentation.

Brynn Abrico

Brynn Abrico

```
def walks(length):
    '''calculates the number of walks of a given length that are possible
    with steps up, down, and right but without immediately doubling back'''
    return walkHelp(length, 'x')

def walkHelp(length, step):
    '''given that the last step was in the step ('x' or 'y') direction,
    returns the number of walks that could follow of the given length'''
    if length < 1: return 1    # the empty set gives us a length-0 walk
    else:
        if step == 'x':
            return 2*walkHelp(length-1, 'y') + walkHelp(length-1, 'x')
        else: return walkHelp(length-1, 'y') + walkHelp(length-1, 'x')

print "n : walks(n)"
print "------------"
for i in range(10):
    print i, ":", walks(i)
```

The result of running this code was

```
n : walks(n)
------------
0 : 1
1 : 3
2 : 7
3 : 17
4 : 41
5 : 99
6 : 239
7 : 577
8 : 1393
9 : 3363
```

By inspection of these values, I hypothesized that the number of walks of a particular length $n \geq 0$ followed the recurrence relation

$$w_0 = 1, \qquad w_1 = 3, \qquad w_n = 2w_{n-1} + w_{n-2}$$

Rubic Success 2:

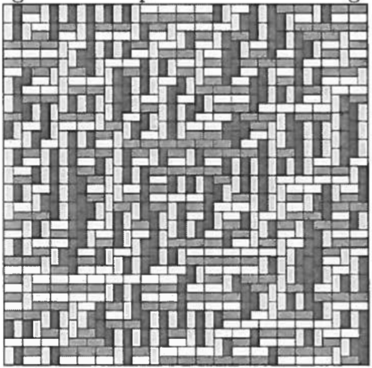
ENCOURAGING MULTIPLE PERSPECTIVES

- Making Connections: "Between Representations"
- Fluidly move between different representations of a mathematical object.

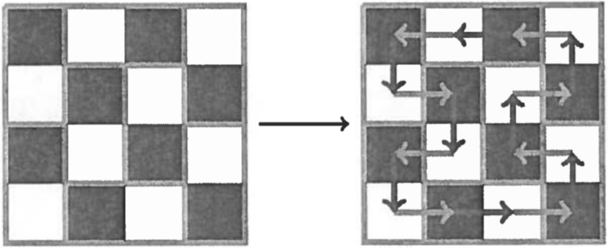
Abram Sanderson

Abram Sanderson

Figure 1: Complicated domino tiling[1]



Abram Sanderson



Abram Sanderson

"Another thing I notice while reflecting upon my work for the "between representations" part of the rubric is that for the majority of the problem, I was constantly switching in my head between [two representations]. I think this switching was ultimately beneficial because even though I ultimately ended up writing up the solution in terms of the original formulation alone, I think the idea of ... is *much* more strong motivated by the [other representation]."

Rubic Success 3:

BALANCING FLEXIBILITY AND PERSEVERANCE

- "Flexibility": Encouraged to play with multiple approaches
- Students embraced this, but valued perseverance as well

Vidushi Ojha

"I tried several different proof techniques and methods of finding a base of substrings, but as soon as the going got tough I switched methods. I think that in the future I should try to stick with a method for a little longer before discarding it, although it is important to recognize when I need to look at something from a new perspective."

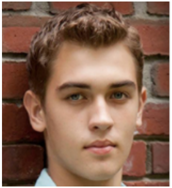
TESTIMONIALS

Testimonial 1

Dylan Baker

"This problem was particularly difficult for me to approach. As I am not a math major, reading math papers is particularly difficult, as often the implied baseline knowledge is slightly higher than my math education research thus far.....however I'm really excited by how much confidence I've gotten out of this semester thus far!"

Testimonial 2



Bryce McLaughlin

"I feel like this project most opened up my ability to question both my own results and the results of others. When my representation and Erdös did not agree I was forced to put both underneath a great deal of scrutiny. In finding flaws in both, I gained a great deal of confidence in my own work by realizing that everything published in a journal is not necessarily an end all fact....this project has given me the confidence that it is not infeasible to be a published scholar."

Collaborators

Gail Tang   Emili Cilli-Turner   Milos Savic

Houssein El-Turkey   Gulden Karakok   David Plaxco

"Pedagogical Practices for Fostering Mathematical Creativity and Proof-Based Courses: Three Case Studies", **RUME 2017**

Thank You!