PEDAGOGICAL PRACTICES THAT FOSTER MATHEMATICAL CREATIVITY AT TERTIARY-LEVEL PROOF-BASED COURSES

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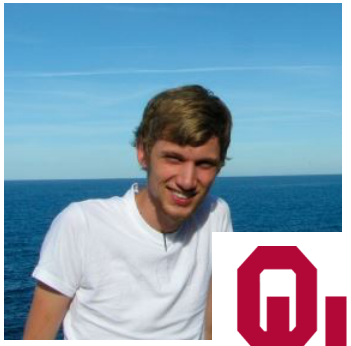
MATHEMATICS

The Creativity Research Group



CPR on Proving

- Creativity-in-Progress Rubric on Proving (Savic et al., 2016; Karakok et al., 2015, 2016; El Turkey et al., under revision)
- Two categories:
  - Taking risks
  - Making Connections
- First created as a research tool, but has morphed into a tool for teacher and student use

CPR on Proving

MAKING CONNECTIONS:

	Beginning	**Developing**	**Advancing**
Between Definitions/Theorems	Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving	Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving	Implements relevant definitions/theorems from the course and/or other resources outside the course in their proving
Between Representations[1]	Provides a representation with no attempts to connect it to another representation	Provides multiple representations and recognizes connections between representations	Provides multiple representations and uses connections between different representations
Between Examples	Generates one or two specific examples with no attempt to connect them	Generates one or two specific examples and recognizes a connection between them	Generates several specific examples and uses the key idea synthesized from their generation

CPR on Proving

TAKING RISKS:

	Beginning	**Developing**	**Advancing**
Tools and Tricks[2]	Uses a tool or trick that is algorithmic or conventional for the course or the student	Uses a tool or trick that is model-based or partly unconventional[3] for the course or the student	Creates a tool or trick that is unconventional for the course or the student
Flexibility[4]	Begins a proof attempt (or more than one proof attempt), but uses only one approach	Acknowledges and/or uses more than one proving approach, but only draws on one proof technique	Uses more than one proof technique
Posing Questions	Recognizes there should be a question asked, but does not pose a question[5]	Poses questions clarifying a statement of a definition or theorem	Poses questions about reasoning within a proof
Evaluation of Proof Attempt	Examines surface-level[6] features of a proof attempt	Examines an entire proof attempt for logical or structural flow	Examines and *revises* an entire proof attempt for logical or structural flow

What is creativity?

- Over 100 definitions (Mann, 2006)
- **A process** of offering new solutions or ideas that are **unexpected** for the student, **with respect to his/her mathematics background** or the problems s/he has seen before (Sriraman & Liljedahl, 2006)

How do we "teach" creativity?

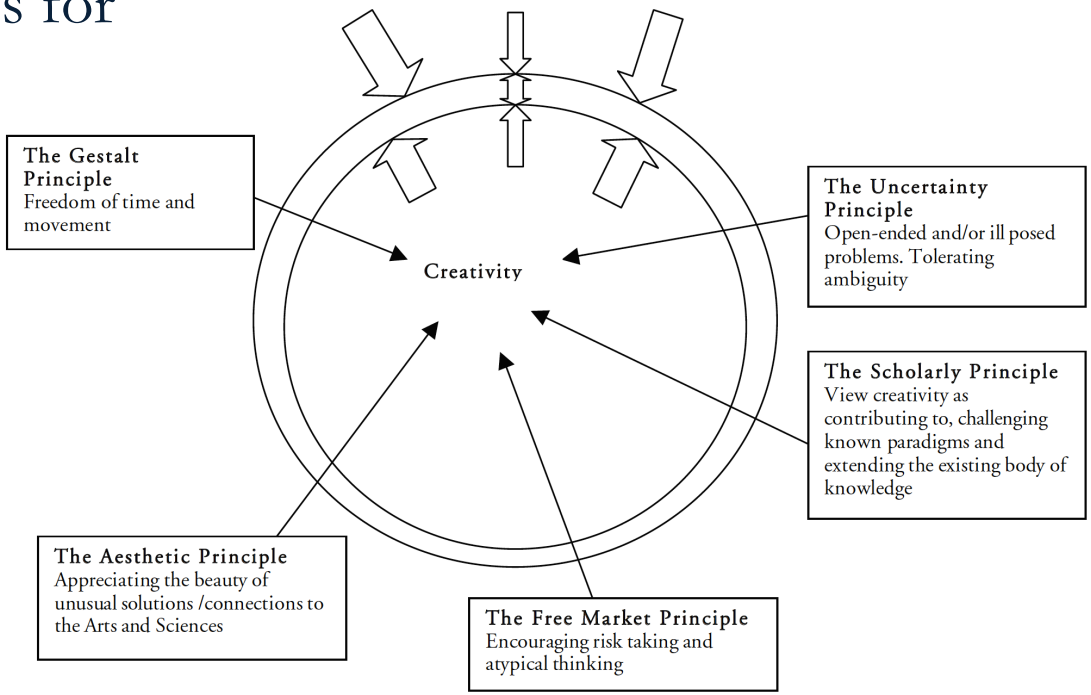
- Mathematical creativity in undergraduate teaching
- From Zazkis and Holton (2009):
  - Learner-generated examples (Watson & Mason, 2005)
  - Counter-examples (Koichu, 2008)
  - Multiple solutions (Leikin 2007, 2009)
  - Changing parameters (Brown & Walter, 1983)
- "At the collegiate level, however, very little empirical research has yet described and analyzed the practices of teachers of mathematics" (Speer, Smith & Horvath, 2010, p. 99)

Teaching Practices – K-12

- Levenson (2011, 2013) – Collaborative creativity
  - choosing appropriate tasks,
  - fostering a safe environment where students can challenge norms without fear of repercussion,
  - playing the role of expert participant by providing a breakdown of the mathematics behind a process, and
  - setting the pace, allowing for incubation periods. (Levenson, 2013, p. 273)

Teaching Practices – K-12

- Sriraman (2005) – 5 principles for maximizing creativity



Five Principles

- Gestalt
  - Opportunities to engage in the four-stage creative process (Wallas, 1926; Hadamard, 1945)
    - Preparation, incubation, insight, verification
- Aesthetic
  - Teacher valuing solutions that utilize unusual proving techniques, come from diverse topics of mathematics, or make efficient or elegant solutions

Five Principles (cont.)

- Free Market
  - Creating a classroom environment that allows students to freely input ideas, thoughts, and solutions
- Scholarly
  - creating a classroom environment "in which students are encouraged to debate and question the validity of… approaches to problems…, be encouraged to generalize the problem and/or the solution, as well as pose a class of analogous problems" (p. 28)
- Uncertainty
  - "Students be exposed to the uncertainty and the difficulty of creating mathematics" (p. 28)

Research Questions

- What teacher actions or practices in the proof-based tertiary classroom might foster students' perceptions of mathematical creativity?
- Do the five principles apply to undergraduate teaching?
- In particular, how does the CPR on Proving enhance the potential for mathematical creativity in a proof-based classroom according to the five principles?

Data Collection

- Three collections of data
  - Teachers' impressions (diaries, goals, interviews)
  - Teachers' instruction (Livescribe™ data, notes)
  - Students' impressions (online survey, interviews, homework)
- Preliminarily coding teacher actions using the five principles, then using student data to corroborate effectiveness of teacher action

Houssein's Class - Setting

- 6 students, Number Thy, Fall 2015
- Houssein explained the rubric in one class period (third week)
- Led discussion by asking about subcategories in CPR
- Had co-constructions of proofs in class, and would refer to the rubric to push proving further
- Students were formally asked to use CPR five times in the course
  - Homework as an evaluation tool
  - During the final for extra credit

Mohamed's Class - Setting

- 18 students, Combinatorics course, Spring 2016
- Active Learning Hybrid
- Used rubric for "portfolio problems"
  - "much more involved, and the intention is to allow freedom to roam with it in any direction you wish."
  - One portfolio problem worth three exercises
  - Minimum 3-page essay summarizing the proving process the students used
  - Open problems in Combinatorics

Gail's Class - Setting

- 14 Students, Transition to proof course, Spring 2016
- Utilized rubric language before introducing CPR
- Reflections throughout the course asking about mathematical creativity
- Showed an attempt from a previous course and attempts from the current students, and had students in group discussion evaluate the students' work using CPR

Uncertainty Principle

- "Mathematics at the professional level is full of uncertainty and ambiguity…Creating, as opposed to learning, requires that students be exposed to the uncertainty and the difficulty of creating mathematics" (Sriraman, 2005, p. 28)

Uncertainty Principle

- Mohamed – Portfolio problems were never graded on completion, but rather progress, and some were open problems
- Gail – Not answering correctness or validity questions right away
- Houssein – Getting stuck in class while proving a theorem on two occasions (having to switch techniques) and discussing this episode with students after completing the proof on the third occasion

Uncertainty – Students

- Mohamed – "You could approach different parts of the problem that **it wasn't about getting an answer as much as just kind of exploring relationships**…this idea that you could actually kind of like create or notice relationships that are like, like in kind of like **new ways rather than just kind of like following a template**…
- Gail – "There were times like 'Well I did this proof, but I'm not sure it's right because of this' and she would respond with 'Well, what do you think class?' And the class would participate in it…And it's just, **using each other and building off of each other** in the class to build what we need, create, **made us creative. It built that creative environment for us.**

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Aesthetic Principle

- Instructors **valuing** solutions that utilize **unusual proving techniques**, come from **diverse topics of mathematics**, or make **efficient** or **elegant solutions**
  - Gail - "...That's the exam 2 'solutions' and I say solutions in quotes because they're not all 100% correct, okay, but it doesn't matter. You know there are still really good ideas in there and that's what I want you to see."
  - Student – "I think tools and tricks especially, because …there's always different ways to go about it… I feel like with the tools and tricks it's hard for any two proofs to be exactly alike."

Free Market Principle

- "…teachers should encourage students to take risks" (Sriraman, 2005, p. 28)
  - Houssein - Asking questions using the rubric's language to encourage students in class
    - Have you made a connection to Theorem T? Or Def A? Have you generated an example to help understand the statement?
  - Student - (On being creative in the course)
    "I would say yes… we were given our books to look back…I found one of the questions on the test to be almost exactly like one of the theorems, **so I related it directly to that theorem**, and it was like two chapters previous, like **no one was thinking that far back**."

Scholarly Principle

(a) (5 pts) Create a theorem where the result would contribute to the mathematical knowledge of our classroom community. OR create a theorem about something that you wondered about while working with sets. **You must use at least two sets.** Other symbols that you might use are "$=$", "$\subseteq$", "$\cap$", "$\cup$", and "$\emptyset$". Feel free to use power sets as well (+2 extra credit points if you do!).

(b) (5 pts) Prove your theorem.

Scholarly Principle

Quiz 5: Conjectures created by class

April 2016

Are the following conjectures true or false? If true, prove. If false, show a counterexample **or discuss why it's false. If truth value cannot be determined because there is missing information, then you may add that missing information. For example, you may additional assumptions. You may also change symbols. For example if it's not true for the two sets to be equal, but it is if one is a subset of each other, then you can change "=" to "⊆".**

1. **Conjecture 1.** Let S and T be sets. Then $P(S) \setminus P(T) \subseteq P(S \setminus T)$.
2. **Conjecture 2.** Let A and B be sets. The $P((A \cap B)^c) = P(A^c \cup B^c)$.
3. **Conjecture 3.** Let A and B be sets. If $A \subseteq B$ and $B \subseteq C$ then $P(C^c) \subseteq P(A^c)$.
4. **Conjecture 4.** Let A and B be sets. If $A \subseteq B$, then $A \subseteq P(B)$.
5. **Conjecture 5.** Let S and T be sets. If $S \subseteq T$, then $T^c \subseteq S^c$.

Scholarly Principle

- Student of Gail - "She wanted to make sure that the only time when she interfered with the process is if really like she didn't really see, like nothing else was going on. But that rarely happened. Usually somebody stepped up. Somebody said something that helped to realize that, 'Wait. What if we did it this way?' And it changed the entire, it just changes everything. It changes the entire proof altogether."

Aesthetic Principle

- Student of Gail – "Miss Tang asked us to tell us something about ourselves, something that we're good at. And one of the guys in class stated that he was lazy, I mean efficient. And so efficient kind of became the way of saying that we did it with less work. We were able to work smarter not harder type of thing."

Pre-Conclusion

- There are signs that using the five principles may shed light on the commonalities of teaching actions between the three case studies
- There is a spectrum of usage for the principles in the classroom
- The CPR on Proving is a "vocabulary package" that might be a tool to utilize the five principles
  - "Tools and tricks"

Future Research

- The CPR allowed students to take a metacognitive look at their own proving process
- "Creative actions might benefit from meta-cognitive skills and vice versa, regarding the knowledge of one's own cognition and the regulation of the creative process" (Katz and Stupel, 2015, p. 69)
- In-depth investigation of one class (Gail)

"So I think that like everyone's capable of mathematical creativity. I think that mathematical creativity is not really kind of taught or not made accessible to people, so I think people a lot of times don't realize that they're capable of being creative."

Thank you!

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Questions to Audience…

- How can you "evaluate" your teaching of soft skills like creativity or metacognition?
- Are there either frameworks or literature that may assist the five principles by Sriraman (2005)?
- Is there a quality component to implementing the five principles? Can one "use" a principle without fostering any creativity from the students?